

SUPPORTING PRODUCTIVE STRUGGLE

TEACHING & LEARNING COLLABORATIVE

PURPOSE

TLCs Mathematics Leadership Academy is focused on creating a cadre of K-8 educators knowledgeable about mathematics education, leadership, facilitation, and quality professional development.

RATIONALE

Across the nation, districts are using classroom teachers to provide leadership in content areas. Resources from the Leadership Academy support teachers as they lead professional learning sessions at the building/district level.

CONTACT US

200 E. Wilson Bridge Rd.
Worthington, OH 43085
614.265.9800



ABOUT THIS RESOURCE

- Thinking about Productive Struggle in Mathematics with K-8 teachers is essential. This resource includes detailed facilitator notes and access to slides which you can use to facilitate a professional development session with K-8 teachers.

FORMAT

- Use as a one-hour session or divide over two 30-minute sessions. Opportunities to engage in a rich mathematical task and continued conversation about the importance of productive struggle and how we can support student learning are included.

MATERIALS NEEDED

- The lesson includes needed attachments.
- Access to the following resource book would be helpful as teachers are asked to read sections from the book.
Note: One is K-5 and one is 6-8.
 - Huinker, D.A., & Bill, V. (2017). Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5. Reston, VA: National Council of Teachers of Mathematics.
 - Smith, M., Steele, M.D., & Raith, M.L., (2017). Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8. Reston, VA: National Council of Teachers of Mathematics.

TEACHINGLEARNINGCOLLABORATIVE.ORG

FACEBOOK.COM/TLCPAGE

TWITTER @CONNECT2TLC

Facilitator Notes

Activity: Supporting Productive Struggle

Leadership Academy 2019

Mathematics Content/Pedagogy Focus Highlight:

- Understand the importance of productive struggle in mathematics instruction
- Gain strategies to support students during productive struggle
- Learn instructional moves to implement and plan for during a task that promotes productive struggle

Prepare Ahead of Time

- Make the quote cubes for each table
- Gather manipulatives
- Copies of needed attachments
- Assemble the productive struggle strips (Attachment D) & quotes (Attachment E)/group
- Access the powerpoint on TLC's website
<http://www.teachinglearningcollaborative.org/k-8-mathematics-leadership-academy.html>

Materials Needed:

- | | |
|--|---|
| <ul style="list-style-type: none"> ● Construction paper ● Colored Pencils ● Patty Paper ● Cuisenaire rods ● Pattern blocks ● Reflection paper or Journal ● Chart paper or large paper ● ½ inch grid paper ● Images for Launch (on ppt) ● Resource Book: <i>Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5 and/or the Grades 6-8 version of the book.</i> | <ul style="list-style-type: none"> ● Quote Cube (Attachment A) ● Task: Joe's Pizzeria (Attachment B) ● Making Connections (Attachment C) ● Productive Struggle Strategy Strips (Attachment D) ● Quotes for the 4 Blocks (Attachment E) ● Productive Struggle Strategies and Indicators (Attachment F) ● Connecting Learning and Instruction (Attachment G) |
|--|---|

Resources/References:

- Huinker, D.A., & Bill, V. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5*. Reston, VA: National Council of Teachers of Mathematics.
- Smith, M., Steele, M.D., & Raith, M.L., (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics.
- Warshauer, H. K. (2015). Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, 20(7), 390. doi: 10.5951/mathteachmidscho.20.7.0390
- NCTM, National Council of Teachers of Mathematics. (2014). *Principles to actions: ensuring mathematical success for all*. Reston, VA.

Conducting the Activity: (1 hour)

Launch: (10 minutes)

1. Have participants roll a quote cube (Attachment A). After rolling the cube, participants should read the quote they rolled, pass it to the next person at the table, and reflect on the quote in their journal or paper as it relates to teaching and learning mathematics.

Facilitator Note: The quotes are labeled on the cube (A, B,C,D, E, F) as a reference point.

2. Have participants keep their reflections in a safe place. You will be revisiting this after the explore part of the lesson. Transition into the launch of the task by sharing with participants: *As we explore a task, I want you to keep in mind your quote and how it relates to teaching and learning mathematics.*
3. Using the powerpoint, show participants the first pizza image (image 1). Ask participants, *“What do you notice?”* and allow participants to share several ideas.
4. Then ask participants, how many slices of pizza are there? How do you know? As participants are sharing how many slices they see and how they determined the total, it is important to record their thinking and ideas on chart paper. Use the guiding questions below to make the mathematics clear and understandable for all. You may want to have copies of each pizza image (from the ppt) taped on chart paper.

Facilitator Note: As you record, be sure to consider a wide arrange of representations, counts, and visuals to showcase mathematical thinking. This is important to model, as it would be in a classroom. Share that seeing both visual and symbolic representations allows students to make the deepest meaning of the content.

Guiding Questions:

- How might you count these?
- How did you determine the total?
- How many slices are there in the whole pizza? How is that different than how many are in the image?

5. Next, show participants the next image (pizza image 2). Ask participants to think about how many slices of pizza are in the image? Record their thinking and ideas on chart paper.

Guiding Questions:

- How did you count to determine how many?
- What is another way to count this pizza?

6. Show participants pizza image 3. Ask participants to think about how the pizza is sliced. Discuss what they notice about this pizza compared to the others. Record their thinking and ideas on chart paper.

Guiding Questions:

- How are these pizza slices different/same to the slices in image 1 and 2?
- How does 1 slice compare to the whole pizza?
- What do you notice about the slices?

Facilitator Note: This image will provide participants to experience how students might compose/decompose and partition a whole. Using student responses, you can gain formative assessment information, to inform instruction on potential misconceptions.

7. Next, show participants the next image (pizza image 4). Use the questions below to guide conversations. Be sure to record their thinking and ideas on chart paper.

Guiding Questions:

- How might you order a pizza with these toppings?
- How could you describe this pizza, using the slices?
- What is another way we could describe how this pizza is made?

Facilitator note: This image allows participants to see how students can begin to make the connections between parts of a whole as it relates to equivalency and comparing fractional parts.

8. Finally, show participants the last image (pizza image 5). Ask participants to think about how many slices of pizza are in the image. Record their thinking and ideas on chart paper.

Guiding Questions:

- How did you determine how many slices?
- How much of the pizza is each type?
- How might you order this type of pizza? What fractional parts might you use?

Facilitator Note: This image provides a context for the task in the explore. This image allows participants to make connections between parts of a whole through iteration. Additionally, how a part can then be composed through operations to determine a larger part of the whole.

Explore: (25 minutes)

1. Share with participants the task from Joe's Pizzeria (Attachment B).

Task: Joe's Pizzeria would like to change their ordering options for customers. They would like customers to have the option to order their pizza toppings specifically to how a pizza could be sliced. The smallest slice in their medium pizza can be sliced

into sixths. Their large pizza can be sliced into eighths. What are the possible combinations a customer could order a medium and large pizza if the smallest piece is a sixth in the medium or an eighth in the large? Each pizza when ordered must have at least one of these slices.

2. Explain to participants as they work to determine how the pizzas could be ordered, they must build a model to show their thinking and then represent their reasoning for each possible pizza order.

Facilitator Note: *Be sure participants build a model first. When representing the whole, they should justify their reasoning to each part of the whole. Encourage participants to not use traditional rules, algorithms, or finding common denominator to operate, rather use relationship between parts of a whole from the model.*

3. After about 15 minutes, pause participants have them reflect in a journal or on paper about how are they feeling while solving the task. Then conduct a discussion, use the guiding questions below. Have participants pause their thinking. It is ok if they have not completed the task. The purpose of the task is to productively struggle as they make sense and reason about the mathematics in the task.

Guiding Questions:

- What was helpful when solving the task?
 - What mathematical connections or concepts were you experiencing/engaging in while solving?
 - How might all students access the learning of this task?
 - During what part of the learning process was difficult? Or students might find challenging?
4. Participants should revisit their reflection from the quote cube and add to their thinking after participating in the task. Then have participants share at their tables or whole group, their quote and reflection regarding productive struggle, teaching and learning in mathematics instruction.
 5. As a whole group, create a 3 column graphic organizer on chart paper and discuss the following in column 1 and 2: (participants can record thinking and ideas shared on graphic organizer)
 - Column 1: What did the facilitators do during the learning process.
 - Column 2: How did you feel during the learning process
 - Column 3: Leave the connections to productive struggle column blank at this point in time

6. Have participants reflect on the 4 Block graphic organizer (Attachment C) with the productive struggle teaching strategies (they do not know what these are at this point in the learning process.) Participants should respond to what connections they can make to the ideas in the graphic organizer and to the learning experience from Joe's Pizzeria.

Duration of the Debrief: (25 minutes)

Debrief Focus: Strategies to Support Productive Struggle in the Classroom

Debriefing the Activity:

1. Using the Grade 1 or Grade 7 task from *Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5 (or the 6-8 version)* have participants read the task and discuss what challenges students might have and what might they do to support those challenges.

Facilitator Note: The Grade 1 task is on page 230 (*The Bead Task*) and the Grade 7 task is on page 182 (*The Deep Dark Secret Task*).

2. Use the Lesson Excerpt from the resource book *Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5 or the Grades 6-8 version of the book*. Have participants read the excerpt and reflect on the following:
 - What do you notice about the teacher and student interactions?
 - What challenges and successes do the students have?
 - What instructional moves does the teacher use to support student learning? What instructional moves does the teacher not use?

Facilitator Note: The Grade 1 teacher-student narrative can be found on pages 232-234 and the Grade 7 teacher-student narrative can be found on pages 183-184.

3. Give each group an envelope that includes a Productive Struggle Strategy strip (Attachment D) and quote (Attachment E) . Be sure that all strategies are distributed. Participants should read their strategy and student indicator. Using the information, have participants go back to the Lesson Excerpt Narrative and determine where in the narrative is the given strategy implemented.
4. On chart paper, participants will create a four-block organizer as shown below, making connections to the narrative, task experience (Joe's Pizzeria), productive struggle strategy strip and the quote in their envelope.

Facilitator Note: As participants are working to identify the productive strategy that is evidenced in their narrative. These ideas can be located in the book *Taking Action: Implementing Effective Mathematics Teaching Practices*.

- Grade 1 connections to these concepts can be found on pages 234- 236
- Grade 7 connections to these concepts can be found on page 185

<p>Strategy Connections to Narrative</p> <ul style="list-style-type: none"> ● What lines in the narrative was the strategy used? How do you know? ● How did the teacher support the strategy? ● How did the student react or respond to the strategy used? 	<p>Strategy Connections to Joe’s Pizzeria Task</p> <ul style="list-style-type: none"> ● When was the strategy evidenced? ● How did the use of the strategy make you feel? ● How did the strategy support productive struggle?
<p>Image or Visual Representation How are you thinking differently about productive struggle?</p>	<p>Productive Struggle Quote and Reflection (use the quote from the envelope)</p>

5. After completing their 4 block graphic organizer on chart paper, participants should go on a gallery walk. As they walk around, encourage participants to visit at least one chart from each of the four strategies.
6. Then revisit the three column chart from the explore phase of the lesson. After learning and discussing strategies to support productive struggle, what connections can we now make to productive struggle. As a whole group, complete column 3, connections to productive struggle.
7. Pass out the Productive Struggle Strategies and Indicators chart (Attachement F) to participants. Have participants compare the chart to their 4 blocks graphic organizer as table groups allow them to discuss, connections they made.
8. If time allows, play the following video, to support the importance of productive struggle in mathematics.

Video: <https://youtu.be/HAd8n5x0LxU>

9. After watching the video, have participants share out what ideas do you want to remember about Productive Struggle when planning or implementing tasks in your mathematics class? Or What was your biggest ah-ha / take-away from today's learning? Keep this or hang it in a place where your participants might see this as a reminder about the importance of productive struggle and how to support students (staff lounge, copy room, hallway, as a page in your lesson plans, etc)

Important Facilitator Notes to Remember:

K-2 Connections:

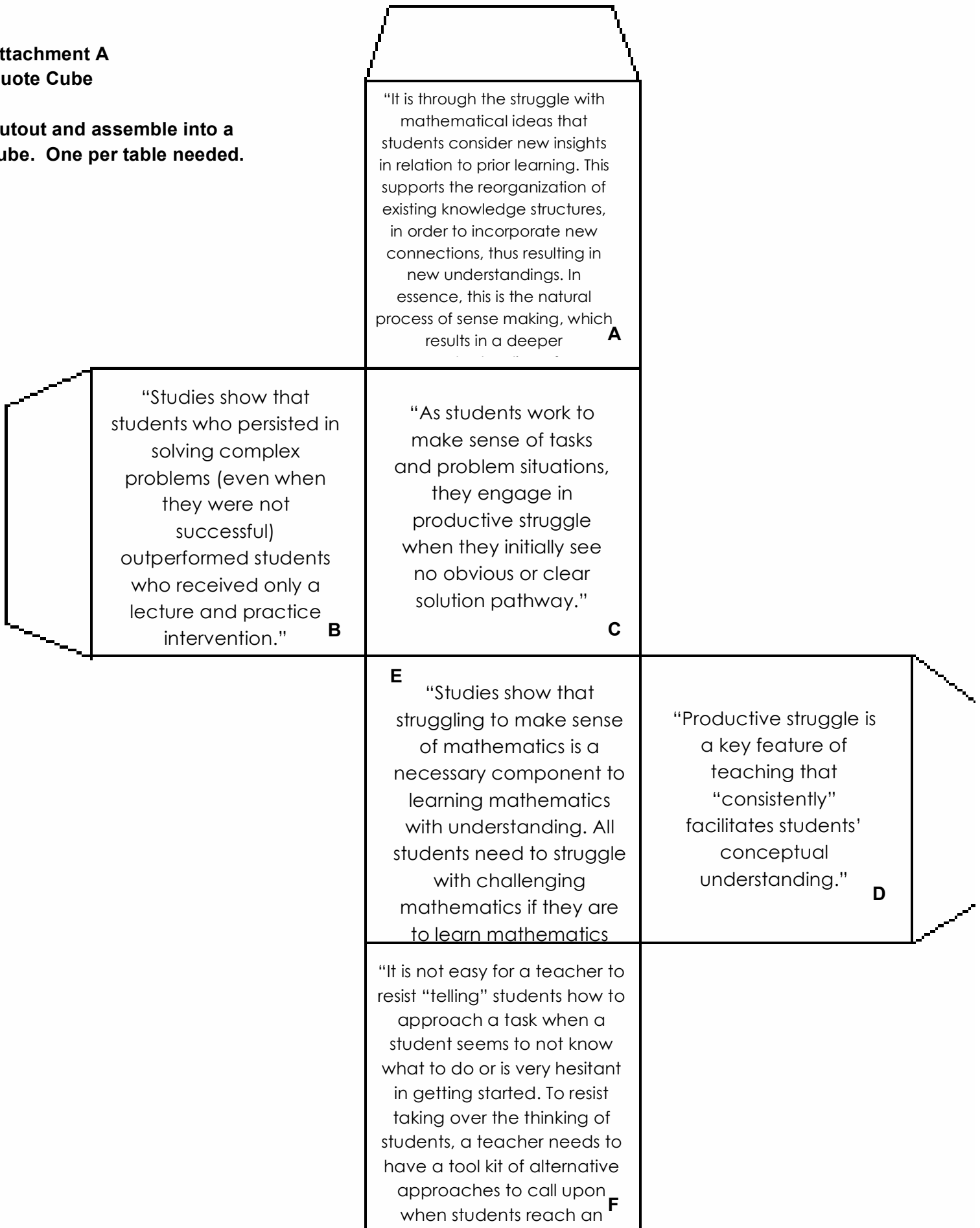
- Students need to understand numbers and shapes can be decomposed and recomposed in many different ways.
- When exploring measurement concepts, students learn to iterate a unit. When working with shapes, the iteration of a unit can also occur. This helps to transfer knowledge of length measurement models to other models and representation where iteration occurs.
- Grade 1 students need to partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of or four of the shares in real-world contexts. Understand for these examples that decomposing into more equal shares creates smaller shares
- Grade 2 students need to partition a rectangle into rows and columns of same-size squares and count to find the total number of them. Additionally, students should partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, or fourths and quarters, and use the phrases half of, third of, or fourth of and quarter of. Describe the whole as two halves, three thirds, or four fourths in real-world contexts. Recognize that equal shares of identical wholes need not have the same shape.

Fractions Concepts Evidenced in Lesson Grades 3-8

- Partitioning
- Equivalence
- Operating
- Comparing Fractions
- Proportional Relationships
- Drawing, Constructing geometrical figures and relationships

Attachment A
Quote Cube

Cutout and assemble into a cube. One per table needed.



Attachment B
(Cut in half and give one to each participant)

Joe's Pizzeria

Joe's Pizzeria would like to change their ordering options for customers. They would like customers to have the option to order their pizza toppings specifically to how a pizza could be sliced. The smallest slice in their medium pizza can be sliced into sixths. Their large pizza can be sliced into eighths. What are the possible combinations a customer could order a medium and large pizza if the smallest piece is a sixth in the medium or an eighth in the large? Each pizza when ordered must have at least one of these slices.

Criteria for Success:

- build a model to show your thinking for each pizza
- represent your reasoning for each possible pizza order

Joe's Pizzeria

Joe's Pizzeria would like to change their ordering options for customers. They would like customers to have the option to order their pizza toppings specifically to how a pizza could be sliced. The smallest slice in their medium pizza can be sliced into sixths. Their large pizza can be sliced into eighths. What are the possible combinations a customer could order a medium and large pizza if the smallest piece is a sixth in the medium or an eighth in the large? Each pizza when ordered must have at least one of these slices.

Criteria for Success:

- build a model to show your thinking for each pizza
- represent your reasoning for each possible pizza order

Attachment C
Making Connections

<u>Ask Question</u>	<u>Give Adequate Time</u>
<u>Acknowledge Student Contribution</u>	<u>Provide Encouragement</u>

Attachment D
Session Materials: Productive Struggle Strategy Strips

	Teaching Strategy	Student Indicators of a Productive Struggle
Question	Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to build on their thinking or look at other ways to approach the problem.	Students ask questions to identify the source of their struggle, write down their ideas, clarify ideas with others, and consider alternative strategies or representations to address their struggle.

	Teaching Strategy	Student Indicators of a Productive Struggle
Encourage	Teachers encourage students to reflect on their work and support students struggle in their effort and not just in getting the correct answer.	Students use their effort to solve problems and try to make sense of their work, not only satisfied with a correct answer or that they perceive themselves as smart or not.

	Teaching Strategy	Student Indicators of a Productive Struggle
Give Time	Teachers give time and support for students to manage their struggles through adversity and failure by not stepping in too soon or too much, thereby taking the intellectual work away from the students.	Students use their time to develop and follow through on their strategies, evaluate their progress, and understand what they can do and what still remains to be done.

	Teaching Strategy	Student Indicators of a Productive Struggle
Acknowledge	Teachers acknowledge that struggle is an important part of learning and doing mathematics.	Students persist in their work, to make sense of and solve their problems and not give up or get discouraged easily.

Attachment E: Quotes for the 4 Block

(Place one strip in an envelope with 1 Strategy and Indicator Strip-from Attachment D)

“When a student cannot get started on a problem, it generally is not because he or she has no relevant knowledge to bring to bear on the situation. More often, the student, for some reason is unable to connect what he or she does know with the current task. A first course of action with the student is to figure out what he or she understands about the problem, by asking questions.”

“Teachers must decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those varied levels of expertise, and how to support students without taking over the process of thinking for them and thus eliminating the challenge.”

“It is often difficult for teachers to see students struggle, but it is through the struggle that learning occurs. Our role as teachers is to provide scaffolds, such as purposeful questioning, that honor and build on the thinking of students without removing the demands of the task or doing the thinking for them.”

“Teachers sometimes perceive student frustration or lack of immediate success as indicators that they have somehow failed their students. As a result, they jump in to “rescue” students by breaking down the task guiding students step by step through the difficulties. Although well intentioned, such rescuing undermines the efforts of students, lowers the cognitive demand of a task, and deprives students of opportunities to engage fully in making sense of the mathematics.”

“One thing to remember, is that the teacher's goal in intervening when a student or group is struggling is not to make sure that every student has a correct and complete response prior to the whole group discussion. Rather, the goal is to “support students' fledgling efforts to make sense of the task before them and to make their thinking headed in a productive direction.”

A good way to recognize if the struggle is productive is to ask yourself, whether the student or the teacher is doing the thinking and the work. The student, when given support should be able to use his or her own effort to move forward in solving the task and understanding the mathematics more deeply.

Struggle is Productive if:

- Intended goals **and** the cognitive demand of the task are maintained
- Student thinking is supported by acknowledging effort and mathematical understanding
- Students are able to move forward in the task execution through student actions

“One challenge teachers often encounter when they engage students in solving high-level tasks is the student (or students) who cannot get started on the task. This problem can sometimes be avoided by selecting a task that has multiple entry points and launching the task so that students are clear on what is happening in the problem and what the teacher is asking them to do.”

Huinker, D.A., & Bill, V. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5*. Reston, VA: National Council of Teachers of Mathematics.

Smith, M., Steele, M.D., & Raith, M.L., (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics.

NCTM, National Council of Teachers of Mathematics. (2014). *Principles to actions: ensuring mathematical success for all*. Reston, VA.

**Attachment F
Productive Struggle
Strategies and Indicators**

	Teaching Strategy	Student Indicators of a Productive Struggle
Question	Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to build on their thinking or look at other ways to approach the problem.	Students ask questions to identify the source of their struggle, write down their ideas, clarify ideas with others, and consider alternative strategies or representations to address their struggle.
Encourage	Teachers encourage students to reflect on their work and support students struggle in their effort and not just in getting the correct answer.	Students use their effort to solve problems and try to make sense of their work, not only satisfied with a correct answer or that they perceive themselves as smart or not.
Give Time	Teachers give time and support for students to manage their struggles through adversity and failure by not stepping in too soon or too much, thereby taking the intellectual work away from the students.	Students use their time to develop and follow through on their strategies, evaluate their progress, and understand what they can do and what still remains to be done.
Acknowledge	Teachers acknowledge that struggle is an important part of learning and doing mathematics.	Students persist in their work, to make sense of and solve their problems and not give up or get discouraged easily.

Warshauer, H. K. (2015). Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, 20(7), 390. doi: 10.5951/mathteachmidscho.20.7.0390

Attachment G
Connecting Learning and Instruction

What did the facilitators do during the learning process?	How did you feel during the learning process?	Connections to Productive Struggle




Teachers are our Superheroes!

Looking for more lessons and resources to engage students and teachers? Check out TLC's SIDEKICK!

sidekick.teachinglearningcollaborative.org

Choose your favorite Superhero and try that lesson!

(Lesson titles on Sidekick are indicated in orange)

	<p>Get your Spidey teacher senses tingling with the “STICKING TOGETHER” lesson. Students make observations about how objects stick based on their environment.</p>
	<p>“A Day in the Life of Mighty Mouse” is a SUPER way to look at plant and animal structures/functions needed for survival and flow of energy within a system.</p>
	<p>“BATS”...need we say more? Students will listen to a story about Echo the Bat to learn about bats, their body structure, how they use echolocation, foods they eat and how/why they migrate.</p>
	<p>“TEN FLASHing FIREFLIES” (see what we did there?) Students listen to the story <i>Ten Flashing Fireflies</i> and as they read they will respond about what they see, combinations of numbers and questions about more and less.</p>
	<p>Check out the Hulk sized “Handful of Peanuts” activity for K-2. Students work to investigate how many of a given object they can grab and work to count and represent this quantity with a model and written number. It would be pretty neat to see how the Hulk would do at this one!</p>

Another SUPER idea is to pull your colleagues together and try some of the Sidekick lessons together! Be sure to let us know how they go, new content ideas you have/need, and suggestions for the site!

Email: sidekick@teachinglearningcollaborative.org