# TASKS THAT PROMOTE REASONING & PROBLEM SOVLING

TEACHING & LEARNING COLLABORATIVE

## PURPOSE

TLCs Mathematics Leadership Academy is focused on creating a cadre of K-8 educators knowledgeable about mathematics education, leadership, facilitation, and quality professional development.

## RATIONALE

Across the nation, districts are using classroom teachers to provide leadership in content areas. Resources from the Leadership Academy support teachers as they lead professional learning sessions at the building/district level.

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## ABOUT THIS RESOURCE

- Engaging students in rich mathematical tasks is critical in the K-8 classroom. This resource includes detailed facilitator notes and access to slides which you can use to facilitate a professional development session with K-8 teachers.

### FORMAT

- Use as a one-hour session or divide over multiple sessions. Opportunities to engage in a rich mathematical task and continued conversation about the importance of using using high-quality tasks in the classroom.

### MATERIALS NEEDED

- The lesson includes needed attachments.
- Access to the following resource book would be helpful as teachers are asked to read sections from the book. Note: One is K-5 and one is 6-8.
  - Huinker, D.A., & Bill, V. (2017). Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5. Reston, VA: National Council of Teachers of Mathematics.
  - Smith, M., Steele, M.D., & Raith, M.L., (2017). Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8. Reston, VA: National Council of Teachers of Mathematics.

# Facilitator Notes
# Activity: Tasks that Promote Reasoning and Problem Solving
Leadership Academy 2019-2020

**Mathematics Content/Pedagogy Focus Highlight:**

- Understand the importance tasks that promote reasoning and problem solving in mathematics instruction
- Discuss and engage in characteristics that determine different cognitive levels for tasks
- Learn how to alter a task to elevate the level of demand

| **Prepare Ahead of Time** |
|---|
| ● Make task analysis cards 1 set for each partnership<br>● Make task analysis label cards 1 set for each partnership<br>● Make task cards 1 set for each partnership<br>● Copy Task cards and Analysis Guide alignment for each partnership<br>● Copy Task analysis guide 1 copy for each participant<br>● Copy rethinking tasks front to back, so each participants has 4.<br>● Copy 1 of each quote on an individual piece of paper |

| **Materials Needed:** | |
|---|---|
| ● Comparing Tasks Attachment<br>● Journals (half sheet of paper or notecards)<br>● Chart Paper<br>● Markers<br>● Quotes<br>● Tape | ● Task Reflection<br>● Task analysis cards<br>● Task analysis label cards<br>● Task cards<br>● Task cards and analysis guide alignment<br>● Task Analysis guide<br>● Rethinking Tasks |

**Resources/References:**


- Huinker, D.A., & Bill, V. (2017). T*aking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5.* Reston, VA: National Council of Teachers of Mathematics.
- Smith, M., Steele, M.D., & Raith, M.L., (2017). T*aking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8.* Reston, VA: National Council of Teachers of Mathematics.
- Warshauer, H. K. (2015). Strategies to Support Productive Struggle. *Mathematics Teaching in the Middle School*, *20*(7), 390. doi: 10.5951/mathteacmiddscho.20.7.0390
- NCTM, National Council of Teachers of Mathematics. (2014). *Principles to actions: ensuring mathematical success for all*. Reston, VA.

**Conducting the Activity: (1 hour)**

**Launch: (20 minutes)**

1. Have participants reflect on implementing tasks and instruction, while thinking about *Tasks that Promote Reasoning and Problem Solving*. Participants should record their reflection or thinking in their journal. You may use some prompts below to support their reflection.
    - What do you consider when implementing tasks?
    - How do you know what task to implement? Or how do you choose?
    - What are successes when implementing tasks?
    - What are challenges when implementing tasks? How do you support these challenges?

2. After reflecting, have participants turn and talk with table partners to share their ideas.

3. Then have groups share whole group. Record participant thinking on chart paper. See the example below.

| **Considerations**<br>(Choosing a Task) | **Implementation**<br>(Frequency) |
|---|---|
| **Successes**<br>(During & After Learning) | **Challenges**<br>(Before, During & After Learning) |

4. Next show participants 2 tasks, *see attachment Comparing Tasks* (from NCTM Taking Action 6-8, pg. 30). In their journals (notecards or half sheet of paper) participants should solve both tasks.

5. As participants finish solving the tasks, they should independently complete the *Task Reflection* graphic organizer.

6. In table groups, participants should share their Task Reflection ideas. After sharing, each group will create a poster, see the model below.

| Similarities between the Tasks | Differences between the Tasks |
|---|---|
| **Features of each Task** | **Explain which task is more likely to promote reasoning and problem solving.** |

7. After participants complete the chart, allow participants to share out their explanations regarding which task is more likely to promote reasoning and problem solving. As participants share record their ideas on chart paper.

8. Share with participants that they will be closely examining Tasks that Promote Reasoning and Problem Solving, what are they, what do they mean for student learning, and what considerations should we take before and during instruction.

**Explore 1: (25 minutes)**

1. Give each partnership a set of Task Analysis cards. Ask participants to reflect on the two tasks *Candy Jar and Missing Values* and determine which statements they believe best align with each of those tasks.

   **Facilitator Note:** Participants might not use all the cards. They may choose to create a category (both) for cards they currently believe relate to both tasks. It is important to allow participants to sort in a way that makes sense to them.

2. After participants have sorted, have participants make a colored dot or circle on the back of the cards for each task category. For example; all statements that aligned with Missing Values gets a blue dot or circle and all statements that aligned with Candy Jar gets a green dot or circle.

3. Then give each partnership a set of Task Analysis Label cards. Explain to participants, "*You have received a bag of label cards. Using all of the cards that describe characteristics of different types of tasks (given in step 1, Task Analysis cards), sort all of these to the category (label card) you feel is the best fit.*"

4. After participants have sorted the cards, use the PowerPoint to slides to facilitate this next part. Show each category one at a time on a slide. As you show each category, have participants revise/edit their category. If they have cards in the category that do not belong, they should discuss where these cards might belong, and move them to that category. *You will continue this for Low Level: Memorization and Low Level: Procedures without Connections.*

5. Now that you have completed two of the four categories, show participants *High Level: Procedures with Connections.* Again, they should revise/edit their category. However, this time if they have cards in the category that do not belong, they should discuss why these might belong in *High Level: Doing Mathematics* (the last category).

6. Next, have participants reflect on the Candy Jar and Missing Values task, looking at the categories and characteristics. Participants should discuss, which type of task, their original statement sort aligned to (using the color circle on the back).

7. Participants should then reflect, to determine what category based off of what they now know, do you the believe each task aligns with.

8. Conduct a whole group discussion regarding their reflections and thinking. Use the guiding questions below to support the discussion.

**Guiding Questions:**

- What characteristics stood out when determining the relationship to either Candy Jar or Missing Values?
- How did solving the two tasks, align with the characteristics?
- What ah-ha's did you have?
- How are you thinking differently about tasks?

**Facilitator Note:**
***Two tasks are a focus of this session and can be found in NCTM Taking Action 6-8 pages 30-33. Please read over the information regarding each task. A few key points have been highlighted below.***

**Candy Jar Task:**

- solution strategy is not implied
- students must solving and justify their thinking.
- all students can access task at their current level
- High Level

**Missing Values Task:**

- directions imply procedures for problem solving
- entry point more challenging
- Low Level

9. Provide each partnership with Task Cards A-J, and a Task Analysis Guide (Page 32-33 of Taking Action, Grades 6-8). Participants should examine and solve tasks A-J and determine how the task engages you with the mathematics. After solving, participants should label the type of task they believe best aligns (*High Level: Doing Mathematics, High Level: Procedures with Connections, Low Level: Procedures without Connections, Low Level: Memorization)*. Participants can use the Task Analysis Guide to help them.

   **Facilitator Note:** You may choose to use tasks from your school district curriculum and/or resources instead of the tasks provided.

10. After participants have determined the categories for each task, give each group the Task Card and Analysis Guide Alignment. Participants should check their thinking and reasoning, with the appropriate corresponding category for each task.

11. Conduct a whole group discussion about Tasks that Promote Reasoning and Problem Solving. Using the guiding questions below to support the discussion.

    **Guiding Questions:**
    - How are you thinking differently about tasks?
    - What wonderings do you have about implementing tasks? (Capture these ideas on chart paper)
    - What new connections did you make about tasks that promote reasoning and problem solving?

**<u>Duration of the Debrief:</u> (30 minutes)**

**Debrief Focus:** Rethinking Tasks and Implementation

**Debriefing the Activity:**

1. In partnerships, participants should choose two of the low-level tasks (A, D, F, and I) and determine how they could change the task to make them a high-level task. All Participants should record their thinking on the Rethinking Tasks graphic organizer.

2. As participants are working to alter the chosen tasks, encourage them to be creative. It is important that they do not make the same change for both tasks.

3. After about 10-12 minutes, have participants take their graphic organizer, stand up and tap ten chairs. When they tap the tenth chair they should sit. Encourage participants to roam around the room so they do not end up with the same colleagues in their new group.

4. Once participants are in their new groups, they should share the task they chose and changes made to elevate the level. While a participant is sharing all group members can record Ah-ha's or Other Ideas they want to remember.

5. After participants shared their ideas and changes to their group, allow participants time to reflect by conducting a quote carousel (one quote a piece of chart paper, there are several quotes in the room). Choose from the list of quotes attached.

6. Explain to participants that they will be reflecting on today's learning, by visiting each quote in the room. As they visit each quote they are to respond and reflect to the quote or another participants response. When responding, they should take a bright color marker with them. Their response should include more than I agree or an exclamation mark, rather a thoughtful response and reflection.

7. After the quote carousel, bring participants together and share the slide *What does this mean for instruction?* Explain to participants that research has shown that the implementation of certain task types and certain instructional moves results in a range of student achieved based on those factors.

8. Then watch a short video from past NCTM President Diane Briars about Tasks.

   Video: https://youtu.be/icHjVR2hC8s

9. After the video, reiterate the last statement from the video and ask participants to consider their learning from today and think about what Diane said in the video; *to what extent do kids have an opportunity to every single day to engage in rich mathematics?* After posing this to participants conduct a discussion. Use the guiding questions to support the discussion.

**Guiding Questions:**

- What is a take away from todays learning?
- How are you thinking differently about tasks?
- What is something you want to change or rethink regarding instruction? Tasks?

**Facilitator Notes to Remember:**

- It is important for participants to understand that there needs to be a balance of cognitive task levels in instruction. It is important, however, to understand how and when these different tasks are implemented. Students should have access to high level tasks frequently, it is through these tasks and discourse that connections can be made to deepen the mathematical understanding. Once students have multiple connected experiences, they can then apply this knowledge to low level experiences.

**Comparing Tasks**

| The Candy Jar Task | The Finding the Missing Value Task |
|---|---|
| A candy jar contains 5 jolly ranchers and 13 jawbreakers. Suppose that you had a new candy jar with the same ratio of jolly ranchers to jawbreakers, but it contains 100 jolly ranchers.<br><br>How many jawbreakers would you have?<br><br>Explain how you know. | Find the value of the unknown in each of the proportions show below.<br><br>$\frac{5}{2} = \frac{y}{10}$<br><br>$\frac{a}{24} = \frac{7}{8}$<br><br>$\frac{n}{8} = \frac{3}{12}$<br><br>$\frac{30}{6} = \frac{b}{7}$<br><br>$\frac{5}{20} = \frac{3}{d}$<br><br>$\frac{3}{x} = \frac{4}{28}$ |

## Task Reflection

| | Candy Jar Task | Missing Value Task |
|---|---|---|
| Strategies that could be used | | |
| Features of the problem | | |

**Task Analysis Cards**

| | | | |
|---|---|---|---|
| Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory. | Are algorithmic. Use of procedure either specifically called for or is evident from prior instruction, experience, or placement of task. | Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. | Require complex and non-algorithmic thinking- a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example. |
| Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. | Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it. | Require some degree of cognitive effort. Although general procedures may be conceptual ideas that under lie the procedures to complete the task successfully and that develop understanding. | Require students to explore and understand the nature of mathematical concepts, processes, or relationships. |

| | | | |
|---|---|---|---|
| Are not ambiguous. Such tasks involve the exact reproduction of previously seen materials, and what is to be reproduced is clearly and directly stated. | Have no connection to the concepts or meaning that underlie the procedure being used. | Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning. | Demand self-monitoring or self-regulation of ones own cognitive processes. |
| Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced. | Are focused on producing correct answers instead of on developing mathematical understanding. | Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas opposed to narrow algorithms that are opaque with respect to underlying concepts. | Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. |

| | | | |
|---|---|---|---|
| | Require no explanations or explanations that focus solely on describing the procedure that was used. | | Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. |
| | | | Require considerable cognitive effort that may involve some level of anxiety for the student because of the unpredictable nature of the solution process required. |

**Task Analysis Label Cards**

| | |
|---|---|
| **Lower Level demands: Memorization** | **Lower Level demands: Procedures without Connections** |
| **Higher Level demands: Procedures with Connections** | **Higher Level demands: Doing Mathematics** |

## Task Cards

- Use the Task Cards A-J (Huinker, D.A., & Bill, V. (2017). T*aking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5*. Reston, VA: National Council of Teachers of Mathematics. **Pages 44-46**

- Cut task cards for use in the session for each group.

## Task Card and Analysis Guide Alignment

(Greater detail of each task is included on Pages 47-48 of Taking Action Grades K-5)

| Cognitive Level | Task Cards | Explanation |
|---|---|---|
| High Level: Doing Mathematics | B and J | • No specific pathway<br>• Student decides approach to the problem<br>• Students may revise the story problem as they work with the task<br>• Students may have to create a context<br>• Task J offers two different types of division problems, where students have to compare these problem types.<br>• Tasks can be solved in more than one way |
| High Level: Procedures with Connections | C, E, G ,H | • Tasks provide a pathway related to conceptual understanding<br>• Students develop deeper understanding of concepts in relationship to procedures<br>• Students show their reasoning and use multiple representations<br>• Students are asked to critique a claim. |
| Low Level: Procedures without Connections | A and D | • Task suggests a procedure has been taught<br>• Focus on producing correct answer<br>• Students are not asked to make a drawing, demonstrate understanding, or provide and explanation.<br>• No opportunity to make sense of the problem |
| Low Level: Memorization | F and I | • Recall of facts, formulas, rules<br>• Success based on giving answer, not on problem solving<br>• Limited or no connection to concepts |

## Task Analysis Guide

A guide for examining the cognitive demand of mathematical tasks (From Smith & Stein 1998, p. 348)

### Lower-level demands: Memorization

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.

### Lower-level demands: Procedures Without Connections

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers instead of on developing mathematical understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used.

### High-level demands: Procedures with Connections

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.

- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

**Higher-level demands: Doing Mathematics**

- Require complex and non-algorithmic thinking - a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyze the task and actively examine task constraints that may limit the possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

A guide for examining the cognitive demand of mathematical tasks (From Smith & Stein 1998, p. 348)

## Quotes for Carousel

*While students need opportunities to engage in all types of tasks, research shows that experience with high-level tasks is critical if we want students to develop capacity to engage in thinking, reasoning and problem solving. (Stein and Lane, 1996)*

*It is important to note, that the use of high-level tasks does not guarantee that its potential will be realized during instruction. The interactions of teachers and students during the implementation of such tasks determine whether or not students actually engage in higher levels of mathematical thinking and reasoning.*

*Even though well-intentioned, sometimes the demands of high-level tasks are reduced by teachers, almost subconsciously, in their attempts to help students move beyond confusions and struggles in order to reach correct solutions.*

*Not all tasks provide the same opportunities for student thinking and learning, teacher need to attend to the cognitive demand of tasks they use with their students. (Hiebert et. Al. 1997)*

*Tasks determine potential of classroom instruction for engaging students in high-level thinking and reasoning.*

*Student learning is related to the tasks students experience. Students who have on-going opportunities to engage in high-level mathematical tasks show greater learning gains than students wo spend the majority of their time engage in procedural tasks. (Boaler and Staples 2008; Heibert and Wearne 1993; Stein and Lane 1996)*

*Teachers need to maintain the cognitive demand of high-level tasks when implementing them with students. Unfortunately, high-level tasks are the most difficult to implement well. As students engage with high-level tasks, teachers often lower the level of thinking in their attempts to support student progress toward a solution. (Stein, Grover and Henningsen 1996; Stigler and Hiebert 2009).*

*High-level tasks no only hold high mathematical expectations for every student, one aspect of equitable classrooms, they also "allow multiple entry points and varied solution strategies. (NCTM 2014, p.17) Thus, tasks that promote reasoning and problem solving provide ways for each and every student to enter into the mathematics and encourage students to demonstrate their knowledge in multiple ways.*

*The mathematics students learn from high-level tasks is significantly different than the mathematics they learn from low-level tasks.*

*It is important to note that not all tasks that promote reasoning and problem solving have to be set in a context or need to consume an entire class period or multiple days. What is critical is that a task provide students with the opportunity to engage actively in reasoning, sense making, and problem solving so that they develop a deep understanding of mathematics.*

*Student learning is greatest in classroom where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where tasks are routinely procedural in nature (Boaler and Staples 209; Hieber and Wearne 193, Stein and Lane 1996).*

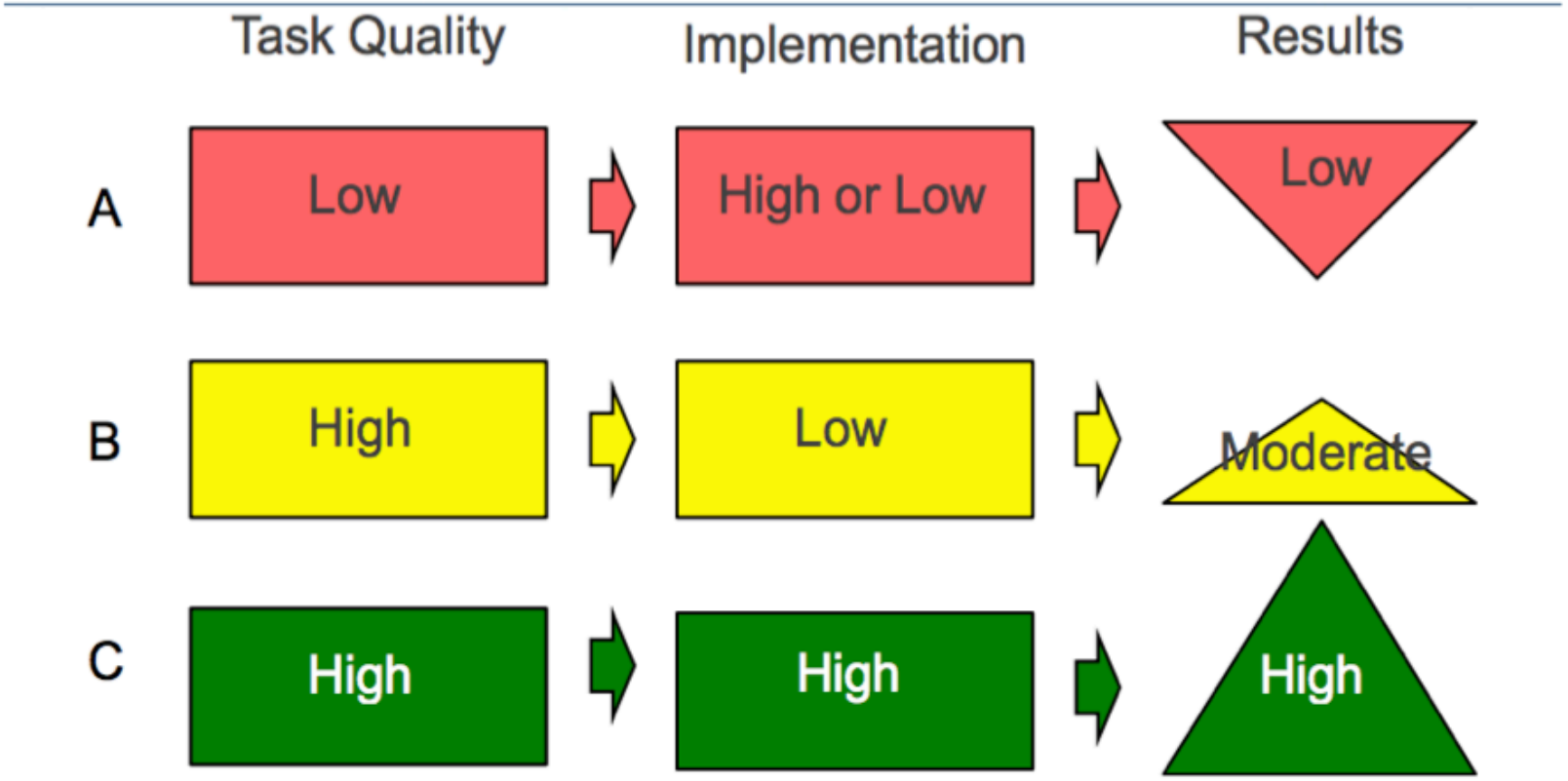
**Rethinking Tasks**

| | Task _____ Level:<br><br>New Level: | Task _____ Level:<br><br>New Level: |
|---|---|---|
| **Changes** | | |
| **Ah-ha's or Other Ideas** | | |

**Rethinking Tasks**

| | Task _____ Level:<br><br>New Level: | Task _____ Level:<br><br>New Level: |
|---|---|---|
| **Changes** | | |
| **Ah-ha's or Other Ideas** | | |

**What does this mean for instruction?**
*(slide)*




Stein, M.K. & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation* 2(4), 50-80.

**Teachers are our Superheroes!**
Looking for more lessons and resources to engage students and teachers? Check out TLC's SIDEKICK!

## sidekick.teachinglearningcollaborative.org

***Choose your favorite Superhero and try that lesson!***
*(Lesson titles on Sidekick are indicated in orange)*

| | |
|---|---|
|  | Get your Spidey teacher senses tingling with the "**STICKING TOGETHER**" lesson. Students make observations about how objects stick based on their environment. |
|  | **"A Day in the Life of Mighty Mouse"** is a SUPER way to look at plant and animal structures/functions needed for survival and flow of energy within a system. |
|  | "**BATS**"…need we say more? Students will listen to a story about Echo the Bat to learn about bats, their body structure, how they use echolocation, foods they eat and how/why they migrate. |
|  | "**TEN FLASHing FIREFLIES**" (see what we did there?) Students listen to the story *Ten Flashing Fireflies* and as they read they will respond about what they see, combinations of numbers and questions about more and less. |
|  | Check out the Hulk sized "**Handful of Peanuts**" activity for K-2. Students work to investigate how many of a given object they can grab and work to count and represent this quantity with a model and written number. It would be pretty neat to see how the Hulk would do at this one! |

Another SUPER idea is to pull your colleagues together and try some of the Sidekick lessons together!
Be sure to let us know how they go, new content ideas you have/need, and suggestions for the site!

**Email: sidekick@teachinglearningcollaborative.org**